SPEED OF SOUND IN A TWO-PHASE MEDIUM WITH SURFACE TENSION

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An improved method of calculating the gas content and speed of sound in a two-phase medium with inclusion of surface tension forces is given.

The method of calculating the speed of sound and gas content in a two-phase medium in [1] takes into account the compliance of the walls of the channel and gives the correct limiting behavior when $\varphi = 0$ and $\varphi = 1$. However, phenomena due to the molecular pressure of the fluid on the gas bubbles is not taken into account. This leads to errors in the determination of the propagation velocity of pressure waves in the two-phase medium. The error is most significant in rapid nonstationary processes in which high-frequency oscillations of the pressure from a certain maximum value up to the saturation pressure of the fluid can occur.

We consider a portion of the channel of a hydrodynamic system in which the gas bubbles are spheres of radius R and are uniformly distributed over the volume of the channel. The pressure P_g inside a bubble is the sum of the pressure P of the liquid and the additional pressure P^* due to the tension of the liquid film surrounding the bubble:

$$P_g = P + P^*. \tag{1}$$

According to the Laplace equation

$$P^* = 2\sigma/R. \tag{2}$$

Assuming that the expansion (or contraction) of the gas bubbles is polytropic, we obtain

$$E_g = nP_g = n(P + 2\sigma/R). \tag{3}$$

By definition (see [1]),

$$V_g = \varphi V, \tag{4}$$

but, on the other hand,

$$V_g = mV_b = \frac{4}{3}\pi m R^3. \tag{5}$$

From (4) and (5),

$$m = 3V\varphi_0/(4\pi R_0^3), \tag{6}$$

where R_0 and φ are the radius of the gas bubble and the gas content, respectively. The latter must be specified for a certain pressure P_0 and temperature t_0.

Using (4)-(6), we have from (3)

$$E_g = n(P + 2\sigma\varphi_0^{1/3}\varphi^{-1/3}/R_0). \tag{7}$$

In this case the solution of the differential equation for the gas content [1, p. 56, Eq. (11)] has the following form, where intermediate calculations are omitted:

$$\varphi = \varphi_0[(1 - \varphi_0)\exp(-f\Phi) + \varphi_0]^{-1}, \tag{8}$$

where

$$f = \exp\{2\sigma(1 - \varphi_0)/[3nR_0(P + 2\sigma/R_0)]\}; \tag{9}$$

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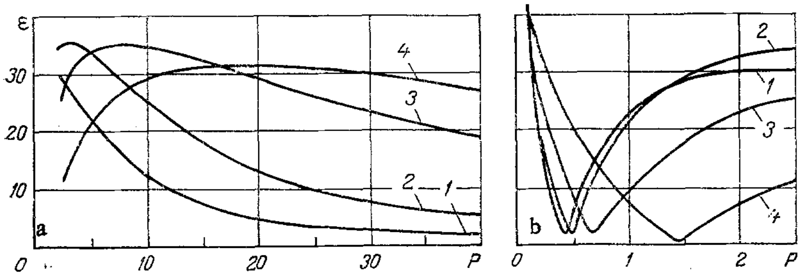

Fig. 1. Dependence of the relative error on pressure for different values of the initial gas content: a) P = 2.5-40 MPa; b) P = 0.1-2.5 MPa. 1) $\varphi_0 = 0.05$; 2) 0.15; 3) 0.5; 4) 0.75. ε, %.

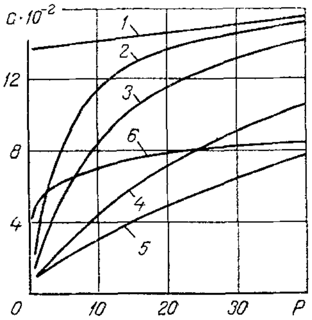

Fig. 2. Dependence of the speed of sound in a two-phase medium with surface tension included for different values of the initial gas content: 1) φ_0 = 0; 2) 0.05; 3) 0.15; 4) 0.5; 5) 0.75; 6) 1.0. P, MPa; c, m/sec.

$$\Phi = \Phi(P) - \Phi(P_0); \tag{10}$$

$$\Phi(P) = \frac{1}{A}\left\{\ln E_l - \sum_{j=1}^{\infty}\frac{a^j}{j!}\left[\ln\frac{E^*}{E_l} - \sum_{i=1}^{j-1} C_{j-1}^i\left(-\frac{E_l}{E^*}\right)^i\frac{1}{i}\right]\right\} - \frac{1}{n}\left[\ln P_g^* - \sum_{j=1}^{\infty}\frac{1}{jj!}\left(-\frac{a_1}{P_g^*}\right)^j\right]; \tag{11}$$

$$a = a_1 A/(E_0 - AP_0^*),\quad a_1 = (1-\varphi_0)P_0^*/3n,\quad P_0^* = 2\sigma/R_0,$$

$$E^* = AP_g^*,\quad P_g^* = P + P_0^*,$$

$$C_{j-1}^i = \begin{cases}(j-1)!/[i!\,(j-1-i)!] & \text{for}\quad i \neq j-1,\\ 1 & \text{for}\quad i = j-1,\\ 0 & \text{for}\quad j-1 = 0.\end{cases} \tag{12}$$

We also have

$$\rho_g = \rho_{g0}\,[(P + 2\sigma\varphi_0^{1/3}\varphi^{-1/3}/R_0)/P_0]^{1/n}, \tag{13}$$

where ρ_{g0} is to be specified at a certain value of the pressure P_0 and temperature t_0.

Equation (8), as in the analogous equation of [1], has the correct limiting behavior when $\varphi_0 = 0$ and $\varphi_0 = 1$.

The series of (11) is summed to a specified accuracy with the help of the Leibniz theorem on the convergence of alternating series; according to this theorem the error in the calculation of the series in (11) is $\varepsilon < |d_j|$, where d_j is the first discarded term of the series.

Calculations of the speed of sound according to [1] with and without (8)-(13) show that the error resulting from not taking into account the surface tension can reach 30-35% for pressures 2.5-15 MPa and 40% for P ≈ 0.1 MPa, and the error grows significantly for pressures less than 0.1 MPa (Fig. 1).

We note that the required number of terms of the series in (11) to attain a relative error of 10^{-4} varies from 2 to a maximum of 4, depending on the input values.

As an example, we show in Fig. 2 the dependence of the speed of sound in a two-phase medium on pressure, taking into account the compliance of the walls of the channel and the surface tension. The different curves refer to different initial values of the gas content for "industrial 20" oil. In the calculations $\sigma = 0.03$ N/m, $R_0 = 0.3$ μm (taken from the data of [2]), and the rest of the data were taken from [1]. The calculations were performed on the SM-4 computer.

NOTATION

φ, gas content; V, V_g, volume of the gas–liquid mixture and the undissolved gas, respectively; σ, surface tension; c, pressure wave propagation velocity; E, modulus of elasticity; ρ, density; E_0, A, empirical constants; n, polytropic index; V_b, volume of a bubble; m, number of bubbles; ε, relative error, in %; subscripts: ℓ, liquid; g, gas; 0, initial state.

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TIME-OF-FLIGHT MASS-SPECTROMETER WITH DUST-IMPACT ION SOURCE

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The processes occurring in a dust-impact mass analyzer, the ion source in which a plasmoid is formed in the impact of a high-velocity dust particle on the anode of the device, are studied.

1. A dust-impact mass analyzer (DIMA) is a combination of a time-of-flight mass spectrometer with a dust-impact ion source (see Fig. 1; a detailed diagram is presented in [1]). The device was developed in connection with the VEGA and Jotto expeditions to Haley's Comet [2]. The source of high-velocity particles is the dust cloud around the comet, moving toward the VEGA space probe with a velocity of $\mathbf{v}_0 = 78$ km/sec. The scientific goal of the project is the determination of the chemical and isotopic compositions of the comet material, which is important for the theory of evolution of the solar system. The device can also be used for mass-spectrometric analysis under terrestrial conditions [3].

The theory reveals two basic drawbacks of the device. The first one is the drop in the throughput capacity C of the device in the case of dust particles with low density ρ_d and mass M, where $C = N_c/N$. The question of the magnitude of C in the case of small dust particles is important, since for small N_c the limited sensitivity of the collector can cause the mass spectrum to be lost.

The drop in C is determined by the fact that the plasmoid formed in an oblique collision of the dust particle with the anode (see Fig. 1) moves at some angle to the normal, and as a result most of the ions fly past the collector. The problem of determining C involves the calculation of the angular distributions. This work is concerned with the problems arising here.

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